Simultaneous Modeling of Internal Erosion and Deformation of Soil Structures

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ABSTRACT: Piping, as the result of internal erosion of embankments, is a primary cause of embankment breaks. Moreover, hollows or cavities of soil structures have been occasionally reported due to erosion within soils. The inner states of embankments are needed to be estimated to take appropriate measures for the prevention of the accidents of soil structures. In this study, simultaneous modeling of the internal erosion and the deformation of soils is carried out. As for the internal erosion, the equations governing the erosion and the transport of fine particles within a soil mass are derived with the concept of erosion rates of soils. Additionally, the constitutive model which can describe the deformation of soils caused by the internal erosion is proposed in this paper.

INTRODUCTION

Piping is the phenomenon that a flow path, where the seepage flow concentrates, appears within soil structures. Usually the flow path is created due to the erosion and the migration of soil particles. Piping is the primary cause of dam breaks. Actually, Foster et al. (2000) investigated world-wide embankment dam failures and accidents, and reported that 46 % of them were triggered by piping. The water leakage from aging irrigation ponds, which are small embankment dams to store irrigation water, has been frequently reported in Japan. This phenomenon is considered to result from piping. Therefore, piping is a serious problem for soil structures subjected to seepage flow.

Up to now, there have been a dozen studies on piping. However, the computational methods for evaluation of piping potential are currently limited. Piping is a phenomenon deeply related with erosion within soils, although soil mechanics, at present, cannot deal with the internal erosion of soils and the transport of eroded soil particles. In order to treat the internal erosion and the transport of eroded fine particles within soils, the following must be considered: (1) the velocity field of the intergranular saturated-unsaturated seepage flow, (2) the increase in the porosity of soils owing to the erosion, and (3) the transportation of the eroded soil particles through the soil pores. In this paper, first, the governing equations for the above three items are introduced with the Eulerian formulation. Secondly, the simultaneous modeling of the internal erosion and the deformation of soils is presented, proposing a constitutive model describing the change of the state surface due to the erosion.

GOVERNING EQUATIONS FOR INTERNAL EROSION

The erosion rate of soils is introduced and the soil phases are considered here in advance of deriving the governing equations. The erosion rate is defined as the volume of the eroded soil particles from the unit surface area of the erodible region within a unit of time, which has the dimensions of velocity. The previous empirical studies and the recent semi-theoretical investigations have adopted the following form for the erosion rate as a function of the shear stress exerted onto the erodible soil particles (e.g., Reddi et al. 2000; Indraratna et al. 2009):

$$E = \alpha(\tau - \tau_c) \tag{1}$$

where E, α, τ, and τ_c denote the erosion rate, the erodibility coefficient, the shear stress, and the critical shear stress, respectively. If shear stress τ, exerted by a fluid, is smaller than critical shear stress τ_c, erosion does not occur.

Figure 1 shows the four phases of soils in order to deal with internal erosion and the transport of the detached soil particles from the soil skeleton. Soils are usually divided into three phases, namely, pore air, pore water, and soil particles. When the internal erosion of soils is considered, however, two types of soil particles exist, i.e., the particles of the soil skeleton and those eroded or detached from the soil fabric.

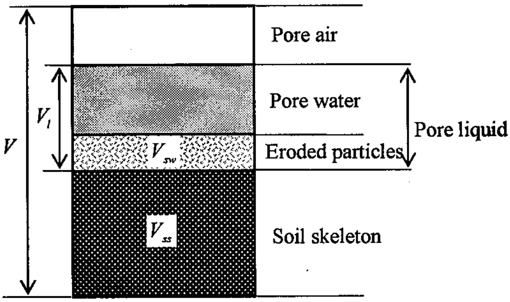

FIG. 1. Four phases of soils for considering internal erosion.

As shown in Figure 1, the mixture of pore water and eroded soil particles is defined as the pore liquid in this paper. V, V_{ss}, V_{sw}, and V_l denote the volume of the soil mass, the soil skeleton, the eroded soil particles, and the pore liquid, respectively. Using these definitions, the soil properties are introduced as follows:

$$\theta = V_l / V \ , \ n = (V - V_{ss}) / V \ , \ C = V_{sw} / V_l \tag{2}$$

where θ, n, and C denote the volumetric liquid content, the porosity, and the concentration of detached soil particles from the soil fabric contained in the pore water, respectively.

Adopting the concept of the erosion rates E and the definition of the soil properties in equation (2), governing equations to analyze the internal erosion of saturated soils are derived as follows:

$$\frac{\partial n}{\partial t}+\frac{\partial v_i}{\partial x_i}=E\,A_e \tag{3}$$

$$\frac{\partial(1-n)}{\partial t}=-E\,A_e \tag{4}$$

$$\frac{\partial nC}{\partial t}+\frac{\partial Cv_i}{\partial x_i}=E\,A_e \tag{5}$$

where t, x_i, n, v_i, A_e and C denote time, Cartesian coordinates, the porosity, the velocity of the pore fluid, the surface area of the erodible region per unit volume and the concentration of soil particles within the pore fluid, respectively. In the above equations (3) to (5), the Einstein summation convention is applied (the summation convention will be applied hereafter with respect to subscripts). The following Darcy's law is applicable to the flow of the pore fluid;

$$v_i=k_s\frac{\partial h}{\partial x_i}\ ,\quad h=z+\frac{u_w}{\rho g} \tag{6}$$

where k_s, h, z, u_w, ρ, g denote the permeability, the hydraulic head, the elevation head, the pressure and the density of pore liquid and the gravitational acceleration, respectively. With the aid of equations (4) and (6), equation (3) can be reduced into

$$\frac{\partial}{\partial x_i}\left(k_s\frac{\partial}{\partial x_i}\left(z+\frac{u_w}{\rho g}\right)\right)=0\ . \tag{7}$$

The shear stress exerted onto erodible soil particles needs to be estimated in order to evaluate the erosion rate of soils, as seen from equation (1). Efforts to estimate the shear stress in the interior of soils have required the idealization of the soil pores as an ensemble of pore tubes. However, the pore tube dimensions distribute within soils and it is quite difficult to determine the spatially distributed values of shear stress. To overcome this difficulty, the representative pore tube dimensions have frequently been used. If the permeability and the porosity of a soil material are given, representative pore diameter $\hat{D}$ is obtained as

$$\hat{D}=4\sqrt{2K/n} \tag{8}$$

where K denotes the intrinsic permeability, defined as

$$K=k_s\,\mu/(\rho g) \tag{9}$$

where μ is the viscosity of the pore liquid. Assuming the Hagen-Poiseulle flow in the pore tube, the shear stress exerted onto the pore wall is estimated by the following equation (e.g., Reddi et al. 2000):

$$\tau=\rho gI\sqrt{2K/n} \tag{10}$$

where I stands for the hydraulic gradient. The erosion rate E can be estimated with equations (1) and (10) when the permeability, the porosity and the hydraulic gradient are determined.

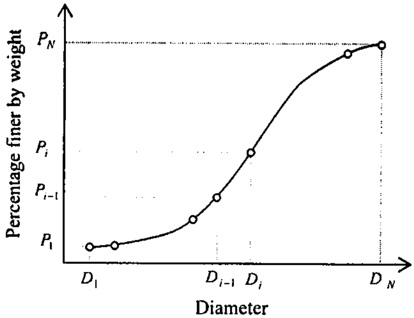

FIG. 2. Illustration of particle distribution curve.

EROSION MODEL

The surface area of erodible soil particles per unit volume of soils, A_e, is required in order to assess how much and how fast the soils erode. However, very few studies have been done on the erodibility within soils and it is considerably difficult to estimate the amount of erodible particles in ordinary soils with a wide range in particle distribution. Therefore, a simple model for the internal erodibility of soils is proposed here in advance of the subsequent numerical analysis.

Supposing a particle distribution curve is given, as shown in Figure 2, the soil particles with a diameter between D_{i-1} and D_i account for the percent of $P_i - P_{i-1}$ by weight. Letting ρ_s denote the density of the soil grains, the particles with a diameter between D_{i-1} and D_i weigh $\rho_s(1-n)(P_i - P_{i-1})$ within the unit volume of the soil, because the soil mass of the unit volume contains the soil particles of $\rho_s(1-n)$ by weight. Using the following averaged particle diameter D_i' as the representative value of the fraction

$$D_i' = (D_{i-1} + D_i)/2 \tag{11}$$

the surface area of the fraction existing within the unit volume of the soil mass, A_e^i, is obtained as follows:

$$A_e^i = \rho_s(1-n)(P_i - P_{i-1})\frac{A_2(D_i')^2}{\rho_s A_3 (D_i')^3} \tag{12}$$

where A_2 and A_3 are the shape coefficients used to calculate the surface area and the volume, respectively. The erodible particles are assumed to be the soil particles which are smaller than the pore size. Considering representative pore size $\hat{D}$, given by equation (8), with porosity n and intrinsic permeability K, the surface area of the erodible particles is determined by the following equation:

$$A_e = \sum_{D_i' < \hat{D}} A_e^i = \sum_{D_i' < \hat{D}} A_2(1-n)(P_i - P_{i-1})/(A_3 D_i') \tag{13}$$

Due to erosion, the material parameters of the density and the viscosity of the pore liquid and the permeability vary. The density and the viscosity of the pore liquid are updated with the following equations:

$$\rho = C\rho_s + (1-C)\rho_w \tag{14}$$

$$\mu = \eta(1 + 2.5C) \tag{15}$$

where ρ_w and η denote the density and the viscosity of pure water, respectively. The permeability varies because of changes in the fluid viscosity, so that it is also updated as follows :

$$k_s = D_r^2 \frac{\rho g}{\mu} \frac{C_T n^3}{(1-n)^3} \tag{16}$$

where C_T and D_r are the material constant and the representative particle diameter, respectively.

REPRODUCTION OF THE EXPERIMENTAL RESULTS

The applicability of the concept presented above is verified here. Reddi et al. (2000) conducted internal erosion tests using a material comprising 70% Ottawa sand and 30% kaolinite clay by weight. Details of the experiments are presented in their paper; thus, only a brief explanation is given here.

Reddi et al. (2000) compacted the test material in the mold and prepared a cylindrical sample with the dimensions of 101.6 mm in diameter and 50 mm in thickness. The sample was saturated with and permeated by distilled water, and the seepage flow in the thickness direction was generated, which caused erosion in the interior of the sample. They measured the turbidity of the effluent, which was converted to the discharge rate of kaolinite. During the experiments, they controlled the inflow rate and recorded the water pressure at the inlet of the sample.

The dashed line in Figure 3 shows the recorded discharge rate of kaolinite. During the tests, the flow rate was increased linearly from 0 to 200 ml/min up to 900 minutes and then kept constant at 200 ml/min after that. Since the pore water pressure was continuously measured, an alteration of the permeability was experimentally obtained during the tests. In order to verify the proposed concept, a reproduction of the experimental data by the numerical simulation was carried out. The numerical results of the kaolinite discharge rate are shown as the solid line in Figure 3. It can be seen that the numerical results provide a good agreement with the experimental data.

The finite element method was used to solve equation (7) and the finite volume approach was applied for equation (5). As the boundary conditions, the flow rate equivalent to their control was imposed on the top and the atmospheric pressure was applied as the pore liquid pressure at the bottom. Integrating the kaolinite discharge rate in Figure 3, with respect to time, the initial surface area of the erodible soil particles was determined by obtaining the total eroded soil mass (=1.1×10^{-3}kg) and multiplying it by the specific surface area of kaolinite (=20×10^3 m^2/kg). Dividing it by the volume of the test sample (=4.05×10^{-4} m^3), the surface area per unit volume A_r was obtained. Other parameters for the gravitational acceleration (=9.8 m/s^2), the density of water (=1000 kg/m^3), the soil particles (=2600 kg/m^3), and the viscosity of pure water (=1.005×10^{-3} kg/m·s) were given, and the critical shear stress (=1.1 Pa) and the initial porosity (=0.27) were provided by the results of Reddi et al. (2000). Only the erodibility coefficient, α, could not be determined. Thus, its value was arbitrarily given as 5.5E-8 m^3/kN·s to match the experimental data shown in Figure 3. Up to now, the direct determination of the erodibility coefficient for internal erosion has not been done due to the great difficulty involved in doing so. Khilar et al. (1985) and Reddi et al. (2000), however, indicated that the erodibility coefficient of internal erosion is even smaller than that of surface erosion, namely, in the range of 1.0E-4 to 1.0E-3 m^3/kN·s. The estimated value of the coefficient, 5.5E-8 m^3/kN·s, is consistent with their indication.

This analysis has shown that the numerical results seen in Figure 3 agree well with the experimental data and that the proposed approach can reproduce the observed data sufficiently well.

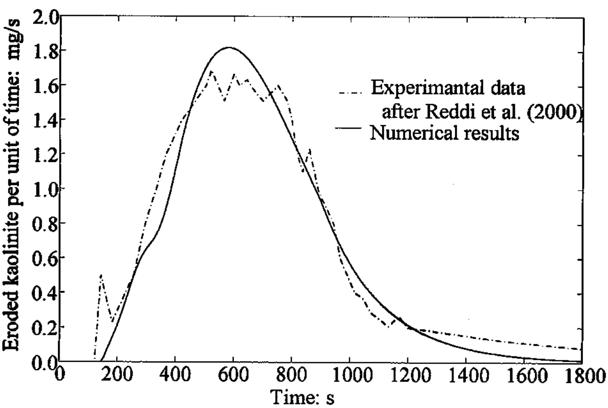

FIG.3. Relationship between discharge rate of kaolinite and elapsed time (after Reddi et al., 2000)

CONSTITUTIVE MODEL TREATING DEFORMATION DUE TO INTERNAL EROSION

Due to the erosion and transport of fine soil particles, the deformation of the soil will be induced. Wood & Maeda (2008) have reported that the internal erosion affects the critical state of the soil. As they indicated, the critical state line moves upward (See Figure 4), which means the soil reaches shear failure at the higher void ratio. Therefore, the failure and the internal erosion are significantly related with each other, and the deformation and erosion of soils need to be simultaneously estimated.

Figure 4 shows the change of CSL (critical state line) and NCL (normal consolidation line) caused by the loss of the fine particles due to the erosion. As seen in the figure, NCL as well as CSL vary the locations due to the erosion and move upward in the conventional semi-logarithmic compression plane (specific volume and logarithm of mean stress). For simplicity, these two lines are assumed to move in parallel, which corresponds to the parallel shift of the state surface. When the increment of the void ratio due to erosion Δe_E is greater than the displacement of NCL and CSL $\Delta\Gamma$, the stress point enters the plastic region and the plastic compression occurs. Dividing the variation of the void ratio Δe into the parts due to the erosion Δe_E and the deformation Δe_D as follows;

$$\Delta e = \Delta e_E + \Delta e_D \ , \tag{17}$$

the following yield function can be obtained on the basis of Cam-clay model.

$$\Delta\Gamma - \Delta e_E - \Delta e_D{}^p = \frac{\lambda - \kappa}{\mathrm{M}}\left(\frac{q}{p'}\right) + (\lambda - \kappa)\ln\frac{p'}{p_0} \tag{18}$$

$$\Delta e_D{}^p = -(1 + e_0)\Delta\varepsilon_v{}^p \tag{19}$$

where λ, κ, M, p', q, $\Delta e_D{}^p$ and $\Delta\varepsilon_v{}^p$ denote the compression and swelling index, the slope of the failure line, the effective mean stress, the deviator stress, the plastic component of the void ratio change and the volumetric plastic strain change. p_0 and e_0 are the reference effective mean stress and void ratio, respectively. This above procedure can be applied to any existing constitutive model based on the critical state concept.Using the yield function of equation (18), the elasto-plastic deformation due to the internal erosion can be calculated by the equilibrium equation of a soil mass. However, it has to be noted that equations (3) to (5) must be modified, as follows, by the Lagrangian formulation considering the deformation of the soil;

$$\dot{n} + nD_{kk} + \frac{\partial v_i}{\partial x_i} = E\,A_e \tag{20}$$

$$-\dot{n} + (1 - n)D_{kk} = -E\,A_v \tag{21}$$

$$(\dot{C\theta}) + C\theta D_{kk} + \frac{\partial C v_i}{\partial x_i} = E\,A_e \tag{22}$$

where D_{ij} denotes the stretching tensor for the deformation of the soil.

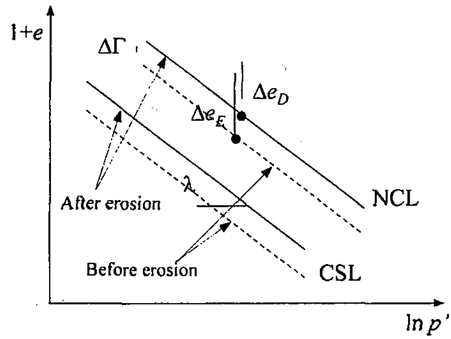

FIG.4. Conceptual diagram of the alteration of the state surface owing to internal erosion (Upward shift of NCL and CSL).

SUMMARY

This paper has presented the simultaneous modeling of the internal erosion and the deformation of soils. In order to describe internal erosion, the concept of the four phases of soils has been introduced. Usage of the erosion rate of soils has enabled the simple derivation of the governing equations for the seepage flow, the changes in porosity, and the transport of the detached soil particles from the soil skeleton. Considering upward shift of the state surface of soils, an example of the constitutive models describing the deformation due to the internal erosion has been proposed on the basis of the Cam-clay model.

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